American Journal of Mathematics and Statistics 2013, 3(4): 237-248
DOI: 10.5923/j.ajms.20130304.08

On the Utilization of Bayesian Approach to Bass Growth Model in Power Consumption Capacity

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Abstract This paper explores the Bayesian technique in evaluating the exterior and interior factors affecting power consumption using the perspective of experts' given a particular level of saturation. The study in Nigeria reveals that T_1 and ψ_1 for external (θ_1) and internal (θ_2) factors have equal annual rate of 0.526 on power consumption. The annual rates of T_2, T_3 *and* T_4 on θ_1 are 0.260, 0.110 and 0.104 respectively, while that of ψ_2, ψ_3 *and* ψ_4 on θ_2 are 0.241, 0.149 and 0.089 respectively and experts have the overall opinion that Nigeria power consumption capacity are influenced by the external factor (θ_1) and internal factor (θ_1) at a rate of 0.25 *and* 0.251 respectively. The external factor (θ_1) effects are overestimated, but the internal factors (θ_2) falls within the purview of the Bass models howing that these factors have almost equal unconditional likelihood effects on the power sector in Nigeria. The growth curves indicate that saturation was achieved after the time t^*=0.0035 with N_{t^*} and f_{t^*} computed as 7.4 and 927.37 respectively. The point of undulation in terms of penetration rate for power consumption in Nigeria was achieved when t=11.

Keywords Power Consumption, Bayesian Technique, Experts, Exterior and Interior Factors, Growth Curve

1. Introduction

Bayesian Approach is based on a conceptually simple collection of ideas, that is, if we are uncertain about the quantity of a parameter, we can quantify our uncertainties as subjective probabilities for the parameter, and also conditional probabilities for observations we might make given the true value of the parameter (likelihood function). When data arrives, Bayes' theorem tells us how to move from our prior probabilities to the new conditional probabilities for the parameter[1]. Decision on external and internal factors affecting the power growth in terms of generation, distribution and consumption growth has caused contestation among a number of historically active social groups in power policy debate. In decision making, multiple perspectives of different individuals or sectoral stakeholders are needed more than ever before, this is particularly true when the decision environment becomes more complex such as accommodating aggregated interior and exterior factors affecting the desired growth in the power sector from the sectoral stakeholders' when limited information are available to evaluating the needed growth as it affect the power generation, distribution and consumption. Hence, this chapter aims at eliciting the sectoral stakeholders opinion using Delphi techniques in the context of influencing external and internal factors affecting the anticipated growth in the power sector.

Delphi techniques used in this study to elicit the sectoral stakeholders opinion incorporates an iterative survey method for eliciting information from sectoral stakeholders[2]. Delphi techniques allows the respondents to reevaluate their responses and it was predicated on the logic that "two heads are better than one"[3]. Delphi techniques are developed to reach a consensus from an expert panel for a complex problem where knowledge/information is limited[4, 5 & 6]. Delphi techniques can be used for a plethora of cases, such as sustainable tourism[7 & 8], human resources development [9], government planning[10], environmental management [11], medicine[12, 13] and strategic management[14], while it is applied to select performance indicators in several fields [15]. Typically, Delphi techniques give sectoral stakeholders panel opportunity, to reconsider their responses and anonymity of the sectoral stakeholders panel is guaranteed. There is no agreement on what a sectoral stakeholder is, as different definitions are proposed[13, 16] and whatever definition is given seems arbitrary[23]. Many authors propose an appropriate size of sectoral stakeholders panel varying from a few to a few hundred experts[17, 18]. However, there is no standard number of sectoral

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Published online at http://journal.sapub.org/ajms

stakeholder[4] as it depends on the nature of the problem[19]. For this study, thirty-five (35) sectoral stakeholders cum key informant in charge of power sector, academicians, customers/consumers and union representatives are selected for two main reasons. They perceive the needs of stakeholders in the power sector better than other types of power consumers/customers and the personal experience of companies' executives could be considered as an important criterion[19]. On the other hand, power sector in Nigeria needs total overhauling and power consumers have higher expectations standards[20] .

In power sector, decision maker usually have some advanced information about the states of nature that can be described in terms of a prior distribution, then the Bayes' approach can be applied to the decision process because it provides another means of defining optimality for decision rules, due to the nature of data in power sector, and, after obtaining the sectora stakeholders' opinion on factors affecting power consumption, the prior distribution can be updated by using more timely information about the probability distribution of the state of nature. Such updated information is called the posterior distribution of θ, given the prior distribution and the data $Y = y$. The posterior distribution of θ is just the conditional distribution of θ, given $Y = y$. A procedure for utilizing the prior distribution to aid in the selection of an action is the Bayes' Criterion. In this study, if (θ, Y) is a discrete bivariate random variable, with joint probability distribution given by $\prod\left(\theta_{i(J)} \mid \underline{y}_{k(J)}\right)$, the random variables θ *and* Y each have marginal distributions. The model given by (equation 5.3) is the prior distributions of Θ, the sequence $\{\tau_i \text{ or } \lambda i\}$ follows a Bernoulli distribution since each factors can have two possible outcomes 0 *and* 1, thus for n_L (aggregating sectoral stakeholder's opinion), we assumed a Binomial distribution (likelihood function) such that; $h\left(y_{k(J)} \mid \theta_{i(J)}\right) = \binom{N_L}{y_k} \theta_i^{y_k} \left(1-\theta_i\right)^{N_L - y_k}$ and for a given $J-th$ sectoral stakeholder classification where zero (0) means no presence of the factor and one (1) means factor presence is confirmed by an sectoral stakeholder's opinion.

The scenario models use the combination of ith measures of influences τ_i *and* λ_i and as such they either "occurs" or "does not occurs" for factors, $\{\underline{\theta}_{(J)} = \theta_1 \text{ and } \theta_2\}$;

$$\underline{\theta}_{(J)} = \begin{cases} \theta_1 = f\left(\tau_i : \forall i = 1,2,...,14 \text{ are measures of external influences}\right) \\ \\ \theta_2 = f\left(\lambda_i : \forall i = 1,2,...,13 \text{ are measures of internal influences}\right) \end{cases}$$

For this study, the probability of each state $\theta_{(J)}$ are computed under the independence of each scenarios (or sectoral stakeholders') and factors influencing the key factors are assessed independently as to whether they occur or not.

The basic scenario probability (external factors (θ_1)) chat is as follows:

$$\left.\begin{array}{cccccccc} \text{Scenario} & \text{Probability} & & & \text{Factors} & & & \\ (\text{Sectoral Stakeholders}) & p_i & \tau_1 & \tau_2 & . & . & . \; . & \tau_n \\ Y = 1(y_1 = 0,1) & p_1 & 1 & 1 & . & . & . \; . & 1 \\ . & . & . & . & & & & . \\ . & . & . & . & & & & . \\ . & . & . & . & & & & . \\ Y = N(y_N = 0,1) & p_N & 0 & 0 & . & . & . \; . & 0 \end{array}\right\} \qquad (1)$$

Where:

$$p_i = probability \text{ of occurrence of } \underline{\theta}_{(1)}(f : \tau_i \forall i = 1,...,n) = \frac{Number \text{ of occurence of } \tau_i}{Total \text{ number of factor explaining } \theta_1} \quad \forall \text{ i=1,...,}n.$$

And the basic scenario probability (internal factors (θ_2)) chat is as follows:

$$\left.\begin{array}{cccccccc} Scenario & \Pr obability & & & Factors & & & \\ (Sectoral\ Stakeholders) & p_i & \lambda_1 & \lambda_2 & . & . & . & . \ \lambda_n \\ Y=1(y_1=0,1) & p_1 & 1 & 1 & . & . & . & . \ 1 \\ . & . & . & . & & & & . \\ . & . & . & . & & & & . \\ . & . & . & . & & & & . \\ Y=N(y_N=0,1) & p_N & 0 & 0 & . & . & . & . \ 0 \end{array}\right\} \quad (2)$$

Where,

$$p_i = probability \text{ of occurrence of } \underline{\theta}_{(1)}(f:\lambda_i \forall i=1,...,n) = \frac{Number \text{ of occurence of } \lambda_i}{Total \text{ number of factor explaining } \theta_2} \quad \forall \text{ i=1,...,}n.$$

Assume that the probability of occurrence of $\tau_i\ or\ \lambda_i$ for either $\theta_1\ or\ \theta_2$ is p for occurrence and q for non-occurrence respectively ; then we derives the prior probability $P(\Theta_J)$ for the key factors using the model:

$$\mathrm{P}(\Theta_J) = \sum_{i=1}^{N} w_{i_J} p_{i_J} \quad (3)$$

Where N is the number of sectoral stakeholders selected in the power sector, w_i is the weight that is to be sum to unity, p_i is obtained from equations (1 & 2) for external and internal factors respectively.

By using the models (3) defined above to compute the prior probabilities and consider a random vector of classified observations on sectoral stakeholder's (Y) opinion, $\underline{y} = (y_i = 1\ (presence \text{ of factor) and } y_i = 0 \text{ (No factor))}$ whose density for a given factor vectors parameters; $\underline{\theta}_{(J)i} = (\theta_{(J)1i}, \theta_{(J)2i})$ is $h(\underline{y} \mid \underline{\theta}_{(J)i})$ referred to as likelihood function. The posterior distribution is computed using the conventional formula:

$$\hat{\Pi}(\theta_{i(J)} \mid \underline{y}_{k(J)}) = \frac{h(\underline{y}_{k(J)} \mid \theta_{i(J)})\mathrm{P}(\Theta_{(J)})}{\sum_{R_\theta} h(\underline{y}_{k(J)} \mid \theta_{i(J)})\mathrm{P}(\Theta_{i(J)})} \quad (4)$$

The Bayes estimators for the key indicators would be obtained using the mean posterior distribution as follows:

$$\hat{\theta}_J = \sum_{R_\theta} \theta_{Ji} \hat{\Pi}(\theta_{Ji} \mid \underline{y}) \quad (5)$$

Then, the posterior distribution of θ given y (factors) in this study is;

$$\prod\left(\theta_{i(J)} \mid \underline{y}_{k(J)}\right) = \frac{L\left(\theta_{i(J)} \mid \underline{y}_{k(J)}\right) p\left(\Theta_{i(J)}\right)}{\sum_{\theta} L\left(\theta_{i(J)} \mid \underline{y}_{k(J)}\right) p\left(\Theta_{i(J)}\right)} \quad (6)$$

Suppose that $Y_1,\ldots,Y_N$ produces a sequence of Bernoulli variates with parameter $\theta_{(J)}$, in this we observed the occurrence or non-occurrence factors $\theta_1,\ldots,\theta_L$, which is a realization of a Bernoulli experiment. The model specified in equation 3 is the prior that does not change over the region for which the likelihood is appreciable, thus, the priors are locally uniform (Box and Tiao,1973,pg23).

2. Model Formulation

The posterior distribution is derived as follows: Let us denote the posterior distribution as $\prod\left(\underline{\theta}_{(J)} \mid \underline{y}_{(J)}\right)$; then by definition we have the general formula as:

$$\prod\left(\theta_{i(J)} \mid y_{k(J)}\right) = \frac{h\left(y_{k(J)} \mid \theta_{i(J)}\right) p\left(\Theta_{i(J)}\right)}{\sum_{R_\theta} h\left(y_{k(J)} \mid \theta_{i(J)}\right) p\left(\Theta_{i(J)}\right)} \quad (7)$$

$\forall$ i=1,...,n_L; k=1,......N (number of experts).

Where:

$h\left(y_{k(J)} \mid \theta_{i(J)}\right)$ *is the likelihood function.*

$p\left(\Theta_{i(J)}\right)$ *is the prior distribution and*

$\sum_{R_\theta} h\left(y_{k(J)} \mid \theta_{i(J)}\right) p(\Theta_{i(J)})$

is the marginal distribution.

Where $p\left(\Theta_{i(J)}\right)$ is as defined in equations (3). The sequence $\{\tau_i\ or\ \lambda_i\}$ follows a Bernoulli distribution since each factor can have two possible values 0 and 1; thus for n_L factors we assume a Binomial distribution such that:

$$h\left(y_{k(J)} \mid \theta_{i(J)}\right) = \binom{N_L}{y_k} \theta_i^{y_k} \left(1-\theta_i\right)^{N_L - y_k} \quad (8)$$

For a given J-th sectoral stakeholders classification where zero (0) means no influence of factor is present and one (1) means factor influence is present.

The posterior distribution in general is therefore derived as;

$$\prod\left(\theta_{i(J)} \mid y_{k(J)}\right)=\frac{h\left(y_{k(J)} \mid \theta_{i(J)}\right)p\left(\Theta_{i(J)}\right)}{\sum_{R_\theta} h\left(y_{k(J)} \mid \theta_{i(J)}\right)p\left(\Theta_{i(J)}\right)},$$

$$=\frac{\binom{N_L}{y_k}\theta_i^{y_k}\left(1-\theta_i\right)^{N_L-y_k} p\left(\Theta_i\right)}{\sum_{R_\theta}\binom{N_L}{y_k}\theta_i^{y_k}\left(1-\theta_i\right)^{N_L-y_k} p\left(\Theta_i\right)},$$

for a given j-th sectoral stakeholder classification.

$$=\frac{\theta_i^{y_k}\left(1-\theta_i\right)^{N_L-y_k} p\left(\Theta_i\right)}{\sum_{R_\theta}\theta_i^{y_k}\left(1-\theta_i\right)^{N_L-y_k} p\left(\Theta_i\right)} \tag{9}$$

Then, the mean posterior distribution is derived as:

$$\hat{\theta}_{(J)}=\sum_{R_\theta}\theta_{i(J)}\prod\left(\theta_{i(J)} \mid y_{k(J)}\right)$$

$$=\sum_{R_\theta}\left[\frac{\theta_i\theta_i^{y_k}\left(1-\theta_i\right)^{N_i-y_k} p\left(\Theta_i\right)}{\sum_{R_\theta}\theta_i^{y_k}\left(1-\theta_i\right)^{N_L-y_k}}\right]$$

$$\hat{\theta}_{(J)}=\sum_{R_y}\left[\frac{\theta_i^{y_k+1}\left(1-\theta_i\right)^{N_L-y_k} p\left(\Theta_i\right)}{\sum_{R_\theta}\theta_i^{y_k}\left(1-\theta_i\right)^{N_L-y_k} p\left(\Theta_i\right)}\right] \tag{10}$$

Specifically, for model defined in (3) we have:

$$\hat{\Theta}_{(J)\text{model}}=\sum_{R_\theta}\left[\frac{\theta_i^{y_k+1}\left(1-\theta_i\right)^{N_L-y_k} w_i p_i}{\sum_{i}\theta_i^{y_k}\left(1-\theta_i\right)^{N_L-y_k} w_i p_i}\right]$$

If $w_i=\frac{1}{N}$, then it reduces to;

$$\hat{\Theta}_{(J)\text{model}}=\sum_{R_\theta}\left[\frac{\theta_i^{y_k+1}\left(1-\theta_i\right)^{N_L-y_k} p_i}{\sum_{R_\theta}\theta_i^{y_k}\left(1-\theta_i\right)^{N_L-y_k} p_i}\right] \tag{11}$$

Of interest in this study is the utilization of the prior distribution given by equation (3) to assist in aiding the selection of an action concerning power consumption called Bayes' criterion. Specifically, we will utilize the prior and posterior distribution to compute the external (θ_1) and internal (θ_2) factors (Given in Table 11) which will subsequently be used in Bass model to determine in power consumption growth for a given m.

3. Empirircal Illustration

In formulating a general framework for making decision as it affect power consumption, the decision maker must choose an action from a set of possible actions. In power sector, the set A *consists* of two points y_0 *and* y_1 corresponding to absence of influencing factor (0) and presence of factor (1). In taking an action the decision maker must be aware of its consequences, which will usually also be a function of the "state of nature" A state of nature k is a representation of the influencing factors i.e. external and internal affecting power consumption growth to which action must be taken. Generally, these factors are possible alternative representations of the physical phenomenon to be studied.

Based on a worldwide set recommended power indicators for sustainable development by[21], the Nigeria power demand and supply projection covering 2005-2030 by[22] and the outcomes and findings of the survey conducted on the factors affecting power consumption growth by this study. The information on external and internal factors (Table 11) affecting the power sectors are also classified into two (2) that is external and internal factors, the fourteen (14) external factors are grouped in order of importance requiring urgent attention, they are thus translated into the prior distribution as follows:

$$\left.\begin{aligned}P\left(T_1=\tau_1,\tau_2,\tau_3/\theta_1\right)&=0.5\\P\left(T_2=\tau_4,\tau_5,\tau_6/\theta_1\right)&=0.25\\P\left(T_3=\tau_7,\tau_8,\tau_9,\tau_{10}/\theta_1\right)&=0.15\\P\left(T_4=\tau_{11},\tau_{12},\tau_{13},\tau_{14}/\theta_1\right)&=0.10\end{aligned}\right\} \tag{12}$$

Similarly, the thirteen (13), internal factors are grouped in order of importance requiring urgent attention. Hence, these factors can be translated into the *prior* distribution thus:

$$\left.\begin{aligned} P(\psi_1 = \tau_1, \tau_2, \tau_3 / \theta_2) &= 0.5 \\ P(\psi_2 = \tau_4, \tau_5, \tau_6 / \theta_2) &= 0.25 \\ P(\psi_3 = \tau_7, \tau_8, \tau_9 / \theta_2) &= 0.15 \\ P(\psi_4 = \tau_{10}, \tau_{11}, \tau_{12}, \tau_{13} / \theta_2) &= 0.10 \end{aligned}\right\} \quad (13)$$

The set of factors that k can assume are denoted by $\tau_i \ \forall \ i = 1, 2, ..., 14$ *and* $\lambda_i \ \forall \ i = 1, 2, ..., 13$ for external factors (θ_1) and internal factors (θ_2) respectively. The probability that $P(T_i \ \forall \ i = 1,2,3,4 | \theta_1) = 0.5, \ 0.25, \ 0.15$ *and* 0.10 and also the probability that $P(\psi_i \ \forall \ i = 1,2,3,4 | \theta_2) = 0.5, \ 0.25, \ 0.15$ *and* 0.10. Then T_i *and* ψ_i are ranked as the most critical, very critical, critical and less critical requiring urgent attention.

The opinions of sectoral stakeholders on the influence of external and internal factors are displayed in tables 1 and 2. The actual proportion of the influencing factors falling into four categories as show in tables 3 and 4 differs slightly from the prior distribution given in tables 5 and 6. The posterior distribution showed in table 7 and 8 are more representative of what to expect in the power sector as revealed by the sectoral stakeholders'.

4. Results and Discussions

The stakeholders' opinion on factors affecting power consumption in Nigeria as shown in Table 11, classified as external (θ_1), internal (θ_2) and showed by equations 12 & 13 requires urgent attention and the results of eliciting stakeholders' opinion using Delphi techniques resulted in the posterior distribution given in Table 9 & 10 for external and internal factors respectively. For the external (θ_1) factors, the results reveals that T_1 (Population total for both urban and rural; GDP per capital and Power prices) contributed 0.526 rates in affecting power consumption. T_2 (Power security of supply; dependency cum power use per capital; local temperature and rainfall combined with pollutant emissions by power users; and end-use power prices with and without tax/subsidy, indigenous power production) contributed 0.260 rates in affecting power consumption. T_3 (Shares of power sectors in GDP value added; manufacturing value added by selected power intensities industries, facilities due to accidents with breakdown to power chain; final power intensity of selected power intensity products, power mix i.e. final power, electricity generation and primary power supply) contributed 0.110 rates in affecting power supply in Nigeria, while, T_4 (Power supply efficiency; income inequalities; rate of deforestation; ratio of daily disposable income/private consumption per capital of 20% poorest population to the prices of electricity and intensity of use of forest resources as renewable power source) contributed 0.104 rates in affecting power consumption in Nigeria.

The internal factors (θ_2) also have effects on power consumption, because ψ_1 (inadequate modern control systems with power generation capacity; insufficient gas for power generation and incomplete implementation of the government reform program) contributed 0.526 rates, ψ_2 (industry and market structure; inappropriate electricity prices; commercial framework to support private investment) contributed 0.241, while ψ_3 (inadequate transmission, obsolete and inefficient transmission and distribution equipment combined with low access to electricity supply) contributed 0.149 and finally ψ_4 (billing and revenue collection; low level of human capacity development; vandalization of equipment, transmission and distribution lines with inadequate study of domestic power requirement) contributed 0.089 in affecting power consumption in Nigeria. Internal factor (θ_2) based on the posterior mean computation contributed 0.251 rates in affecting power consumption.

The unconditional likelihood of influencing factors classified as external (θ_1) and internal (θ_2) affecting power consumption in Nigeria occurred at time $t = 11$ given in Table 11, and as shown by growth curve in figures 1 to 4, the most important part of these curves is that the changes are fast and close to point of inflection. Because in general, inflection point is a point on the curve at which the sign of the curvature changes and this may be stationary point but are not local maximal or local minima. For power consumption, an inflection point is critical in time, since it determines whether the next phase of power consumption will be one growth, stagnation or decline. In this study, growth is recorded immediately after the point of inflection t^* = 0.0035, when N_{t^*} are 7.4, 16, 20 and 32 and when f_{t^*} are 927.37, 2000.8, 2501 and 4001. The cumulative effects of these factors on power consumption should attract the primary attention of power stakeholders which should be centered on the peak and decline pattern of these growth curves[25].

5. Conclusions

In this research study we proposed the usage of Bayesian techniques for the quantification of the frequency and severity distributions of eliciting sectoral stakeholders opinions on external (θ_1) and internal (θ_2) factors affecting the growth in power sectors in terms of power generation/distribution/consumption and sales. The method is based on specifying the prior distributions for the parameters on the frequency and severity distributions of these factors using sectoral stakeholders opinions. Then, the prior distributions are weighted with the actual sectoral stakeholders opinions to estimate the posterior distributions of the model parameters. These are used to estimate the mean posterior distribution for the external and internal factors affecting the power sector in Nigeria. The estimation of mean posterior distribution has several appealing features such as: stable estimators, and the ability to take into account sectoral stakeholders opinions on factors affecting growth in the power sector. There are other aspects of the Bayesian techniques that are useful for modeling sectoral stakeholders opinions, such as the hierarchical Bayesian approach which can be used to estimate the prior distribution by combining several sectoral stakeholders opinions on external and internal factors. As a whole, the mean posterior as expressed by the experts' on external factor contributed 0.25 rates in affecting power consumption in Nigeria. This reveals that power consumption capacity in Nigeria are influences by the external factor (θ_1) and internal factor θ_1 at a rate of $0.25\ and\ 0.251$ respectively, showing that these factors have equal influencing effects on the power sector, and hence, requires urgent attention, the diffusion growth curve also revealed the penetration rate of these factors. The sectoral/stakeholder are of the opinion that external factors affecting the power sector in Nigeria exhibit relatively higher effect on the sector compared to internal effects.

APPENDIX

Computation of the posterior distribution for external and internal factors affecting power consumption in Nigeria

Table 1. Frequency of External Factors Classifications

Sectoral stakeholder's opinion classification on factors	τ_1	τ_1	τ_1	τ_2	τ_2	τ_2	τ_3	τ_3	τ_3	τ_3	τ_4	τ_4	τ_4	τ_4
No (0)	$\left(\frac{22}{35}\right)$	$\left(\frac{16}{35}\right)$	$\left(\frac{11}{35}\right)$	$\left(\frac{20}{35}\right)$	$\left(\frac{19}{35}\right)$	$\left(\frac{7}{35}\right)$	$\left(\frac{16}{35}\right)$	$\left(\frac{22}{35}\right)$	$\left(\frac{17}{35}\right)$	$\left(\frac{20}{35}\right)$	$\left(\frac{16}{35}\right)$	$\left(\frac{15}{35}\right)$	$\left(\frac{12}{35}\right)$	$\left(\frac{13}{35}\right)$
Yes (1)	$\left(\frac{13}{35}\right)$	$\left(\frac{19}{35}\right)$	$\left(\frac{24}{35}\right)$	$\left(\frac{15}{35}\right)$	$\left(\frac{16}{35}\right)$	$\left(\frac{28}{35}\right)$	$\left(\frac{19}{35}\right)$	$\left(\frac{13}{35}\right)$	$\left(\frac{18}{35}\right)$	$\left(\frac{15}{35}\right)$	$\left(\frac{19}{35}\right)$	$\left(\frac{20}{35}\right)$	$\left(\frac{23}{35}\right)$	$\left(\frac{22}{35}\right)$

Table 2. Frequency of Internal Factors Classifications

Sectoral stakholder's opinion classification on factors	λ_1	λ_1	λ_1	λ_2	λ_2	λ_2	λ_3	λ_3	λ_3	λ_4	λ_4	λ_4	λ_4
No (0)	$\left(\frac{15}{35}\right)$	$\left(\frac{16}{35}\right)$	$\left(\frac{18}{35}\right)$	$\left(\frac{12}{35}\right)$	$\left(\frac{18}{35}\right)$	$\left(\frac{20}{35}\right)$	$\left(\frac{10}{35}\right)$	$\left(\frac{17}{35}\right)$	$\left(\frac{11}{35}\right)$	$\left(\frac{18}{35}\right)$	$\left(\frac{12}{35}\right)$	$\left(\frac{19}{35}\right)$	$\left(\frac{20}{35}\right)$
Yes (1)	$\left(\frac{20}{35}\right)$	$\left(\frac{19}{35}\right)$	$\left(\frac{17}{35}\right)$	$\left(\frac{23}{35}\right)$	$\left(\frac{17}{35}\right)$	$\left(\frac{15}{35}\right)$	$\left(\frac{25}{35}\right)$	$\left(\frac{18}{35}\right)$	$\left(\frac{24}{35}\right)$	$\left(\frac{17}{35}\right)$	$\left(\frac{23}{35}\right)$	$\left(\frac{16}{35}\right)$	$\left(\frac{15}{35}\right)$

Table 3. Aggregation of External Factors Classification based on equation (12)

Aggregation of Stakeholder 's opinion	T_1	T_2	T_3	T_4
$No\ (0)$	$\frac{49}{105}$	$\frac{46}{105}$	$\frac{75}{140}$	$\frac{56}{140}$
$Yes\ (1)$	$\frac{56}{105}$	$\frac{59}{105}$	$\frac{65}{140}$	$\frac{84}{140}$

Table 4. Aggregation Internal Factors Classification based on equation (13)

Aggregation of Stakeholder 's opinion	ψ_1	ψ_2	ψ_3	ψ_4
$No\ (0)$	$\frac{49}{105}$	$\frac{50}{105}$	$\frac{38}{105}$	$\frac{69}{140}$
$Yes\ (1)$	$\frac{56}{105}$	$\frac{55}{105}$	$\frac{67}{105}$	$\frac{71}{140}$

Table 5. External Factors Classification with their corresponding Prior Distribution

Prior Distribution	0.17	0.08	0.04	0.03	$\sum_{R_{\theta_J}} h\left(y_{k(J)} \mid \theta_{i(J)}\right) p\left(\theta_{i(J)}\right)$
	$h\left(y_{k(J)} \mid \theta_{i(J)}\right) p(\theta_{i(J)})$				
Y \ K	τ_1	τ_2	τ_3	τ_4	
$No\ (0)$	0.079	0.035	0.021	0.012	0.147
$Yes\ (1)$	0.091	0.045	0.019	0.019	0.173

Table 6. Internal Factors Classification with their corresponding Prior Distribution

Prior Distribution $P(\Theta_J)$	0.17	0.08	0.04	0.03	$\sum_{R_{\theta_J}} h\left(y_{k(J)} \mid \theta_{i(J)}\right) p\left(\theta_{i(J)}\right)$
	$h\left(y_{k(J)} \mid \theta_{i(J)}\right) p(\theta_{i(J)})$				
Y \ K	λ_1	λ_2	λ_3	λ_4	
$No\ (0)$	0.079	0.038	0.015	0.015	0.147
$Yes\ (1)$	0.091	0.042	0.026	0.015	0.174

Table 7. Computation of Posterior Distribution (External Factors)

Prior Distribution $P(\Theta_J)$	0.17	0.08	0.04	0.03	$\Pi\left(\theta_{i(J)} \mid y_{k(J)}\right)$
	$\dfrac{\binom{N_L}{y_k}\theta_i^{y_k}\left(1-\theta_i\right)^{N_L-y_k} p\left(\Theta_i\right)}{\sum_{R_\theta}\binom{N_L}{y_k}\theta_i^{y_k}\left(1-\theta_i\right)^{N_L-y_k} p\left(\Theta_i\right)}$				
Y \ K	τ_1	τ_2	τ_3	τ_4	
$No\ (0)$	0.537	0.238	0.143	0.082	1.000
$Yes\ (1)$	0.526	0.260	0.110	0.104	1.000

Table 8. Computation of Posterior Distribution (Internal Factors)

Prior Distribution $P(\Theta_J)$	0.17	0.08	0.04	0.03	$\Pi\left(\theta_{i(J)} \mid y_{k(J)}\right)$
	$\dfrac{\binom{N_L}{y_k}\theta_i^{y_k}\left(1-\theta_i\right)^{N_L-y_k}\, p\left(\Theta_i\right)}{\sum_{R_\theta}\binom{N_L}{y_k}\theta_i^{y_k}\left(1-\theta_i\right)^{N_L-y_k}\, p\left(\Theta_i\right)}$				
k / y	λ_1	λ_2	λ_3	λ_4	
No (0)	0.537	0.259	0.102	0.102	1.000
Yes (1)	0.523	0.241	0.149	0.086	0.999

Table 9. Tabular algorithm for computation of Posterior Distribution (External Factors)

Prior Distribution $P(\Theta_J)$	(a)				(b)					(c) Posterior distribution of θ_1				(d) Mean Posterior Of θ_1
	0.17	0.08	0.04	0.03										
	$h\left(y_{k(J)} \mid \theta_{i(J)}\right)$				$h\left(y_{k(J)} \mid \theta_{i(J)}\right) p(\theta_{i(J)})$				SUM	$\Pi\left(\theta_{i(J)} \mid y_{k(J)}\right)$				
k / y	τ_1	τ_2	τ_3	τ_4	τ_1	τ_2	τ_3	τ_4		τ_1	τ_2	τ_3	τ_4	
No (0)	$\frac{49}{105}$	$\frac{46}{105}$	$\frac{75}{140}$	$\frac{56}{140}$	0.079	0.035	0.021	0.012	0.147	0.537	0.238	0.143	0.082	0.25
Yes (1)	$\frac{56}{105}$	$\frac{59}{105}$	$\frac{65}{140}$	$\frac{84}{140}$	0.091	0.045	0.019	0.018	0.173	0.526	0.260	0.110	0.104	0.25

Table 10. Tabular algorithm for computation of Posterior Distribution (Internal Factors)

Prior Distribution $P(\Theta_J)$	(a)				(b)					(c) Posterior distribution of θ_2				(d) Mean Posterior of θ_2
	0.17	0.08	0.04	0.03										
	$h\left(y_{k(J)} \mid \theta_{i(J)}\right)$				$h\left(y_{k(J)} \mid \theta_{i(J)}\right) p(\theta_{i(J)})$				SUM	$\Pi\left(\theta_{i(J)} \mid y_{k(J)}\right)$				
y \ k	ψ_1	ψ_2	ψ_3	ψ_4	ψ_1	ψ_2	ψ_3	ψ_4		ψ_1	ψ_2	ψ_3	ψ_4	
No (0)	$\frac{49}{105}$	$\frac{50}{105}$	$\frac{38}{105}$	$\frac{69}{140}$	0.079	0.038	0.015	0.015	0.147	0.537	0.259	0.102	0.102	0.25
Yes (1)	$\frac{56}{105}$	$\frac{55}{105}$	$\frac{67}{105}$	$\frac{71}{140}$	0.091	0.042	0.026	0.015	0.174	0.523	0.241	0.149	0.089	0.25

Table 11. Classification of factors affecting power sector according to internal and external factors

External factors (θ_1)	*Internal factors* (θ_2)
1. Population: Total for urban and rural. 2. GDP per capital 3. Power price 4. Power security of supply, installed capacity and power dependency cum power use per capital. 5. Local temperature and rainfall, and pollutant emissions by power users. 6. End-use power prices with and without tax/subsidy, indigenous power production. 7. Shares of power sectors in GDP value added 8. Manufacturing value added by selected power intensive industries. Facilities due to accidents with breakdown to power chains. 9. Final power intensity of selected power intensity products 10. Power mix: final power, electricity generation, and primary power supply 11. Power supply efficiency 12. Income inequalities 13. Ratio of daily disposable income/private consumption per capital of 20% poorest population to the prices of electricity. 14. Intensity of use of forest resources as renewable power source, and rate of deforestation.	1. Inadequate modern control systems and power generation capacity. 2. Insufficient Gas for power generation. 3. Incomplete implementation of the reform program. 4. Industry and market structure. 5. Inappropriate Electricity Pricing. 6. Commercial framework to support private investments. 7. Inadequate transmission. 8. Obsolete and inefficient transmission and distribution equipment. 9. Components Breakdown. 10. Billing and revenue collection. 11. Low level of human capacity development. 12. Maintenance issues and equipment, a transmission and distribution line damages. 13. Inadequate study of domestic power requirements.

Table 12. Computation of point of inflection for sectoral stakeholers' opinion on external (θ_1) and internal (θ_2) factors affecting power consumption in Nigeria

$m\left\{\begin{matrix}Saturation/\\ maturity\ \text{level}\end{matrix}\right\}$	θ_1	θ_2	θ_2/θ_1 *ratio*	*po*int of Infections		
				$N\left(t^*\right)$	t^*	$f\left(t^*\right)$
3,700	0.25	0.251	1.004	7.4	0.0035	927.37
8,000	0.25	0.251	1.004	16	0.0035	2000.8
10,000	0.25	0.251	1.004	20	0.0035	2501
16,000	0.25	0.251	1.004	32	0.0035	4001.6

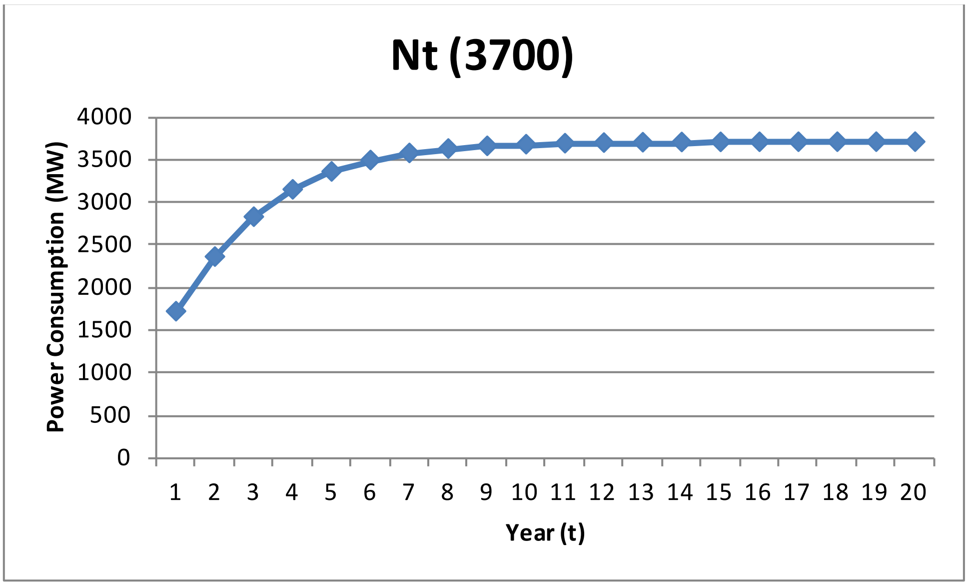

Figure 1. The diffusion Curve Capturing the Sectoral stakeholders' Opinion on External and Internal Factors affecting the Power Consumption Capacity in Nigeria when $m = 3700$ MW

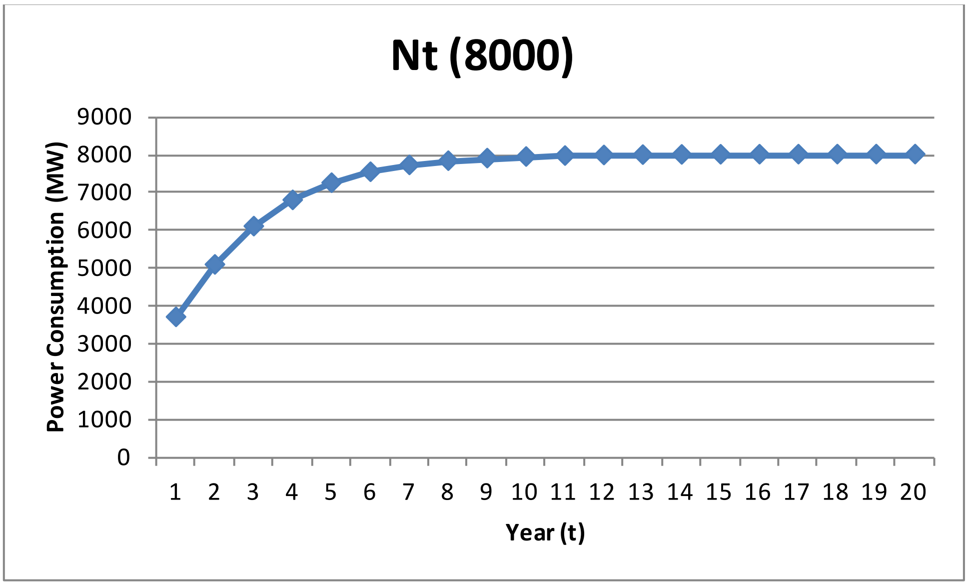

Figure 2. The diffusion Curve Capturing the Sectoral stakeholders' Opinion on External and Internal Factors affecting the Power Consumption Capacity in Nigeria when $m = 8000$ MW

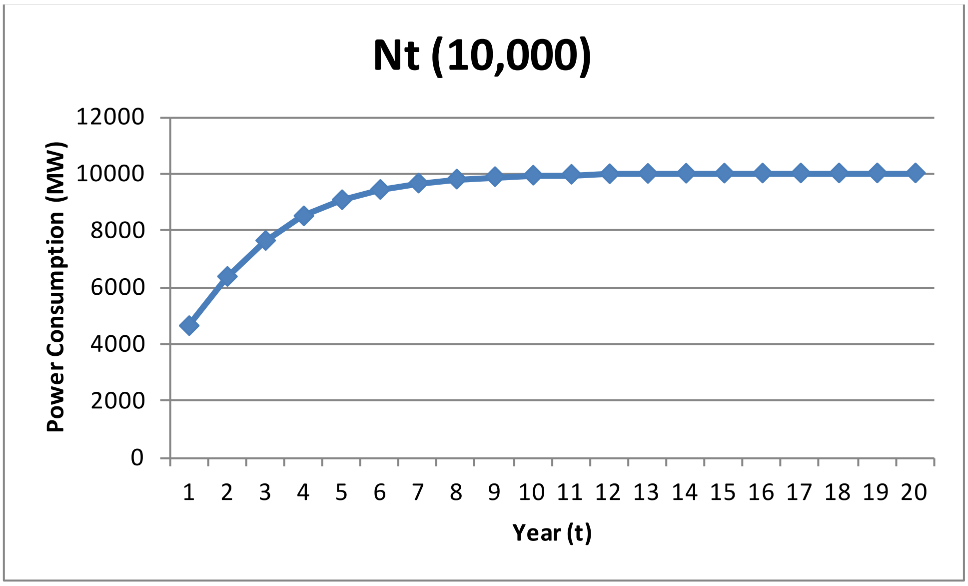

Figure 3. The diffusion Curve Capturing the Sectoral stakeholders' Opinion on External and Internal Factors affecting the Power Consumption Capacity in Nigeria when $m = 10{,}000$ MW

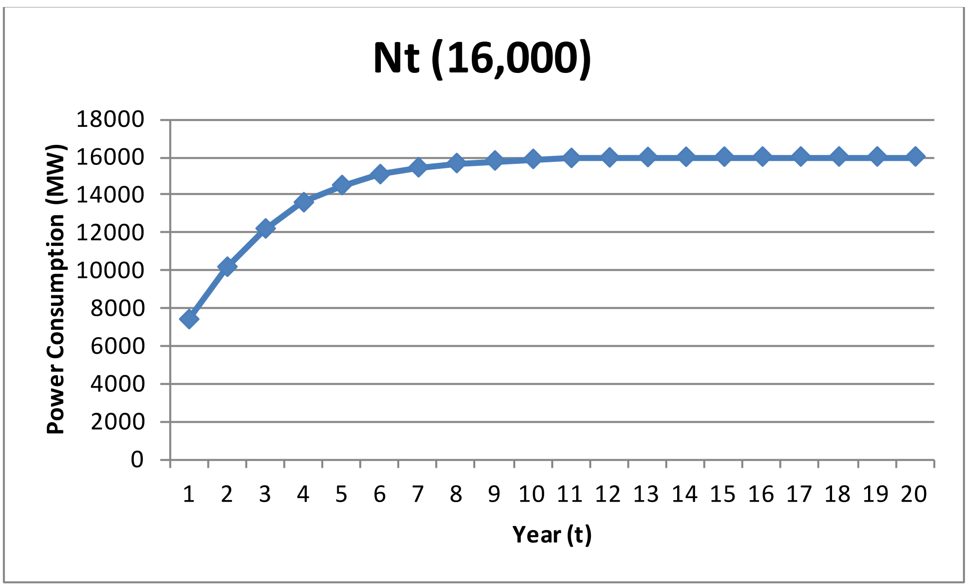

Figure 4. The diffusion Curve Capturing the Sectoral stakeholders' Opinion on External and Internal Factors affecting the Power Consumption Capacity in Nigeria when $m = 16{,}000$ MW

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